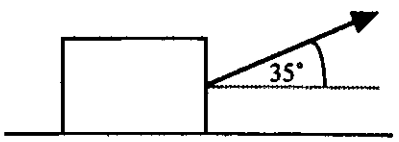
MECHANICS (A) UNIT 1 — TEST PAPER 7

Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. Briefly define the following terms used in modelling in Mechanics:
   (i) lamina, (ii) uniform rod, (iii) smooth surface, (iv) particle. **(4 marks)**

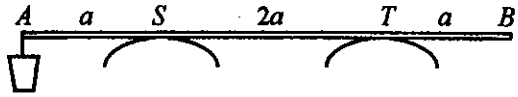
2. Two forces **F** and **G** are given by $\mathbf{F} = (6\mathbf{i} - 5\mathbf{j})$ N, $\mathbf{G} = (3\mathbf{i} + 17\mathbf{j})$ N, where **i** and **j** are unit vectors in the x and y directions respectively and the unit of length on each axis is 1 cm.
   (a) Find the magnitude of **R**, the resultant of **F** and **G**. **(3 marks)**
   (b) Find the angle between the direction of **R** and the positive x-axis. **(2 marks)**
   R acts through the point $P(-4, 3)$. O is the origin $(0, 0)$.
   (c) Use the fact that OP is perpendicular to the line of action of **R** to calculate the moment of **R** about an axis through the origin and perpendicular to the x-y plane. **(3 marks)**

3. A string is attached to a packing case of mass 12 kg, which is at rest on a rough horizontal plane. When a force of magnitude 50 N is applied at the other end of the string, and the string makes an angle of 35° with the vertical as shown, the case is on the point of moving.



   (a) Find the coefficient of friction between the case and the plane. **(5 marks)**
   The force is now increased, with the string at the same angle, and the case starts to move along the plane with constant acceleration, reaching a speed of 2 ms^{-1} after 4 seconds.
   (b) Find the magnitude of the new force. **(5 marks)**
   (c) State any modelling assumptions you have made about the case and the string. **(2 marks)**

4. A uniform yoke AB, of mass 4 kg and length $4a$ m, rests on the shoulders S and T of two oxen.



   $AS = TB = a$ m. A bucket of mass x kg is suspended from A.
   (a) Show that the vertical force on the yoke at T has magnitude $(2 - \frac{1}{2}x)g$ N and find, in terms of x and g, the vertical force on the yoke at S. **(7 marks)**
   (b) If the ratio of these vertical forces is 5 : 1, find the value of x. **(3 marks)**
   (c) Find the maximum value of x for which the yoke will remain horizontal. **(2 marks)**

5. Two small smooth spheres A and B, of equal radius but masses m kg and km kg respectively, where $k > 1$, move towards each other along a straight line and collide directly. Immediately before the collision, A has speed 5 m s^{-1} and B has speed 3 m s^{-1}. In the collision, the impulse exerted by A on B has magnitude $7km$ Ns.

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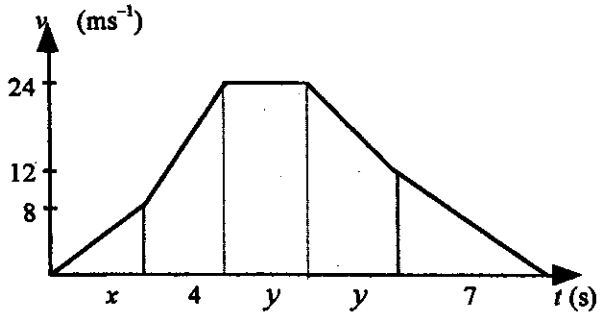
(a) Find the speed of B after the impact. **(3 marks)**

(b) Show that the speed of A immediately after the collision is $(7k - 5)$ ms^{-1} and deduce that the direction of A's motion is reversed. **(5 marks)**

B is now given a further impulse of magnitude mu Ns, as a result of which a second collision between it and A occurs.

(c) Show that $u > k(7k - 1)$. **(4 marks)**

6. 

The velocity-time graph illustrates the motion of a particle which accelerates from rest to 8 ms^{-1} in x seconds and then to 24 ms^{-1} in a further 4 seconds. It then travels at a constant speed for another y seconds before decelerating to 12 ms^{-1} over the next y seconds and then to rest in the final 7 seconds of its motion.

Given that the total distance travelled by the particle is 496 m,

(a) show that $2x + 21y = 195$. **(4 marks)**

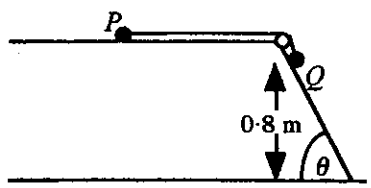
Given also that the average speed of the particle during its motion is 15·5 ms^{-1},

(b) show that $x + 2y = 21$. **(3 marks)**

(c) Hence find the values of x and y, **(3 marks)**

(d) Write down the acceleration for each section of the motion. **(3 marks)**

7. Two particles P and Q, of masses $2m$ and $3m$ respectively, are connected by a light string. Initially, P is at rest on a smooth horizontal table. The string passes over a small smooth pulley and Q rests on a rough plane inclined at an angle θ to the horizontal, where $\tan\theta = \frac{4}{3}$. The coefficient of friction between Q and the inclined plane is $\frac{1}{6}$.



The system is released from rest with Q at a distance of 0·8 metres above a horizontal floor.

(a) Show that the acceleration of P and Q is $\frac{21g}{50}$, stating a modelling assumption which you must make to ensure that both particles have the same acceleration. **(7 marks)**

(b) Find the speed with which Q hits the floor. **(2 marks)**

After Q hits the floor and does not rebound, P moves a further 0·2 m until it hits the pulley.

(c) Find the total time after the system is released before P hits the pulley. **(5 marks)**

